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Dynamic Behavior of Underwater Towed-cable in Linear Profile

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Abstract— In this paper, a numerical approach is presented which is capable of predicting dynamic behavior of underwater towed-cable structures when tow-ship changes its speed in a fixed direction making linear profile. A three-dimensional model of underwater towed system is studied. The governing equations for the system are solved by using a central finite-difference method. The solution of the finite-difference form of the assembled of non-linear algebraic equations is obtained by Newton's method. Since the underwater towed cable model uses implicit time integration, it is stable for large time steps and is an effective algorithm for simulation of large-scale underwater towed systems. The solution of this problem is of practical importance in the estimation of dynamic loading and motion, and thus has direct application to the enhancement of safety and the effectiveness of the offshore activities.

Keywords — Underwater towed cable array; cable dynamics; towed systems; towing manoeuvres; cable tension; numerical simulation; linear profile

1 INTRODUCTION

UNDERWATER towed systems are widely used for many marine applications (naval defense, oceanographic and geophysical measurements etc.). In naval applications, it is used for acoustic detection of submerged targets. In geophysical applications, it is used for oil-prospecting. These systems can be as simple as a single cable with its towed vehicle, or they may be composed of multiple towed cables and multiple towed bodies. A typical component of a towing system is shown in Fig 1. It is well known that the equations of motion for the cable and towed vehicle are non-linear and their dynamic behaviors during various operations are mutually dependent. As a result, these equations are strongly coupled. In order to study the complete problem, they must be solved simultaneously as a whole. It is not easy to solve such a complicated problem analytically and hence numerical methods are usually employed. The most prevalent approaches used in determining the hydrodynamic performance of a cable in an underwater towed system are the lumped mass method [1] and the finite difference method [2]-[10]. However, according to [5] the explicit time domain integration scheme used in the lumped mass method made the method conditionally stable. Burgess [6] pointed out that the time integration used in this algorithm requires the time step to be chosen so that the Courant-Friedrichs-Levy wave condition is satisfied for the highest natural frequency of the lumped mass system. This restricts the use of very small time steps. However, Thomas and Hearn [7] believed that the collapse of the numerical procedure at large time steps in the method is not due to the instability of the numerical scheme, but is caused by the failure of the Newton- Raphson iterative procedure adopted to determine the correct tension levels to solve the nonlinear equations of motion.

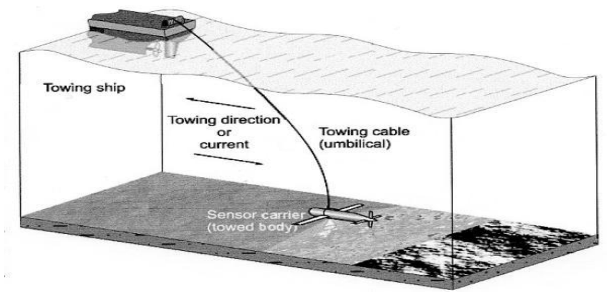

Fig 1: Components of a towing system.

The reason for the collapse of the numerical procedure in the lumped mass method may not be clear, however it is true that time steps in this method must be chosen very small in order to avoid the failure in numerical procedure on the basis of experiences (Burgess [6]; Thomas and Hearn [7]). In the finite difference method, the governing equations for the underwater cable are derived from the balance of forces at a point of cable. Among various finite difference methods, the model developed by Ablow and Schechter [2] is worthy to note. In this model, the cable is treated as a long thin flexible circular cylinder in arbitrary motion. It is assumed that the dynamics of cable are determined by gravity, hydrodynamic loading and inertial forces. The governing equations are formulated in a local tangential-normal coordinate frame which has the un-

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stretched distance along the cable. The differential equations are then approximated by finite difference equations centered in time and in space. By solving the equations, the motion of underwater cable can be determined in the time domain. The principal advantage of this method is that it uses implicit time integration and is stable for large time step sizes. It is a good algorithm for simulation of large-scale underwater cable motion.

In this paper a three-dimensional hydrodynamic model to simulate an underwater towed system is presented. In the model, the governing equations of cable are established based on the method of Ablow and Schechter [2]. The six degrees-of-freedom equations of motion for submarine simulations are adopted to predict the hydrodynamic performance of a towed vehicle. The established governing equations are then solved using a central finite difference method. The solution of finite-difference form of the assembly of non-linear algebraic equations is obtained by the Newton's method. Gauss elimination with partial pivoting is applied to solve the linear system obtained by Newton's method. Since, the model uses implicit time integration; it is stable for large time steps. It also gives more flexibility in choosing different time steps for different manoeuvering problems, and is an effective algorithm for the simulation of a large-scale towed system.

2 Mathematical Model

A mathematical model of manoeuvring of underwater towed cable array system [11] is used to find out the location and tension at any point on the cable as a function of time. The system is treated to be moving under the action of gravity, tow-ship, hydrodynamic loading and inertia forces. The loading function is taken to be the sum of independently operating normal and tangential drags. All these forces are shown in Fig 2.

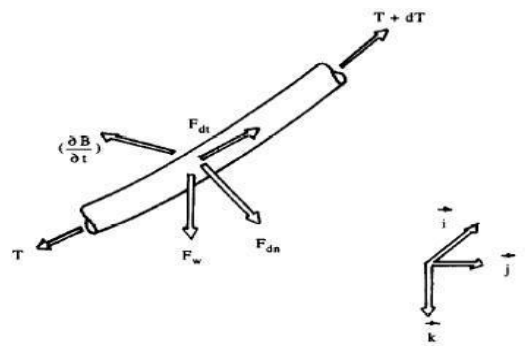

Fig 2: Forces Considered on a Strained System Element of length dS .

The dynamic model is a finite difference approximation to the three dimensional differential equations for conservation of momentum. The total length of the cable-array system is discretized into a number of segments of arbitary length. The time is divided into a number of intervals and various parameters are evaluated at all spatial grid points s_j and temporal grid points t_i .

The dynamic problem formulation is obtained by applying Newton's second law of motion to the cable element of infinitesimaly stretched length dS .

$$\frac{\partial \vec{B}}{\partial t} = \frac{\partial \vec{T}}{\partial S} + \vec{F}_w + \vec{F}_d ,$$

where $\vec{B}$ is the momentum per unit length, $\vec{T}$ is the tension, $\vec{F}_w$ is the weight minus buoyancy per unit length and $\vec{F}_d$ is the force exerted by the fluid on the cable-array system per unit length and is taken to be the sum of independently operating normal drag and tangential. A system of three scale equations is obtained by separating the three components of vector equation in the independent directions $(\vec{t}, \vec{n}, \vec{b})$. The cable orientation is given in the Fig 3.

The compatibility relations in terms of velocity are

$$\frac{\partial}{\partial t}\left(\frac{\partial \vec{r}}{\partial s}\right) = \frac{\partial}{\partial s}\left(\frac{\partial \vec{r}}{\partial t}\right), \qquad (1)$$

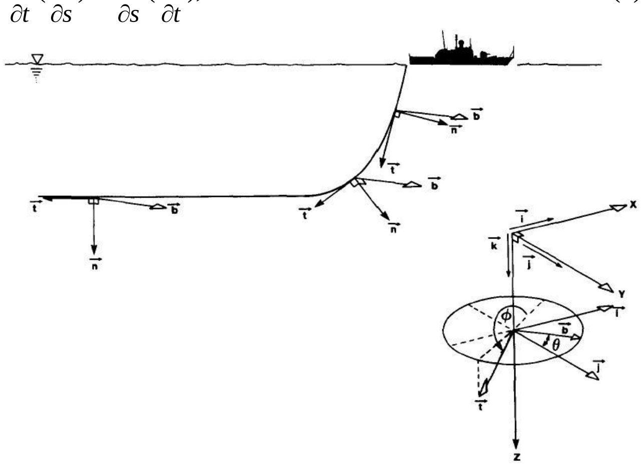

Fig 3: The Orientation of the Cable.

where $\vec{r}$ is a position vector from the origin of a fixed coordinate system $(\vec{i}, \vec{j}, \vec{k})$ to a point on the cable-array system. $\vec{r}$ is a function of unstretched cable-array system-length coordinate s and the time t. By separating various components of the equation (1) in independent directions $(\vec{t}, \vec{n}, \vec{b})$, a system of three scalar equations of compatibility is obtained. Three equations of motion and three equations of compatibility together present six scalar dynamic differential equations of first order in space variable s and time variable t .

The six governing equations of motion in matrix form are

$$M \frac{\partial \vec{y}}{\partial s} = N \frac{\partial \vec{y}}{\partial t} + \vec{q} \qquad (2)$$

where $\vec{y}(s,t)=\begin{pmatrix} T \\ V_t \\ V_n \\ V_b \\ \theta \\ \phi \end{pmatrix}$

$$M(\vec{y},s,t)=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & V_b\cos\phi & -V_n \\ 0 & 0 & 1 & 0 & -V_b\sin\phi & V_t \\ 0 & 0 & 0 & 1 & V_n\sin\phi - V_t\cos\phi & 0 \\ 0 & 0 & 0 & 0 & -T\cos\phi & 0 \\ 0 & 0 & 0 & 0 & 0 & T \end{bmatrix}$$

$N(\vec{y},s,t)=$

$$\begin{bmatrix} -me\frac{V_t}{1+eT} & m & 0 & 0 & (m_1V_b-\rho AJ_b)\cos\phi & \rho AJ_n-m_1V_n \\ e & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1+eT \\ 0 & 0 & 0 & 0 & -(1+eT)\cos\phi & 0 \\ -e\frac{(m_1V_b-\rho AJ_b)}{1+eT} & 0 & 0 & m_1 & (m_1V_n-\rho AJ_n)\sin\phi-m_1V_t\cos\phi & 0 \\ -e\frac{(m_1V_n-\rho AJ_n)}{1+eT} & 0 & m_1 & 0 & -(m_1V_b-\rho AJ_b)\sin\phi & m_1V_t \end{bmatrix}$$

$$q(\vec{y},s,t)=\begin{bmatrix} w\sin\phi+\frac{1}{2}\rho d(1+eT)^{\frac{1}{2}}\pi C_tU_t\,|U_t| \\ 0 \\ 0 \\ 0 \\ \frac{1}{2}\rho d(1+eT)^{\frac{1}{2}}C_nU_n(U_n^2+U_t^2)^{\frac{1}{2}}-\rho A\dot{J}_b \\ \frac{1}{2}\rho d(1+eT)^{\frac{1}{2}}C_nU_n(U_n^2+U_t^2)^{\frac{1}{2}}-\rho A\dot{J}_n \end{bmatrix}$$

where A is the cross section area of unstretched cable; C_n and C_t are normal and tangential drag coefficients; d is diameter of cable; ρ is fluid density; dS is infinitesimal stretched cable length; $e=\frac{1}{EA}$, E is Young's modulus; m is mass per unit length of cable; $m_1=m+\rho A$ is virtual mass per unit length; $w=(m-\rho A)g$ is immersed weight per unit length; g is gravitational acceleration; T is cable tension magnitude; $\vec{v}$ is velocity of tow-ship; $\vec{J}=(J_t,J_n,J_b)$ is current velocity given in local frame $(\vec{t},\vec{n},\vec{b})$; $\dot{\vec{J}}=(\dot{J}_t,\dot{J}_n,\dot{J}_b)$ is the partial derivative of $\vec{J}$ w. r. t. time t holding s fixed; $\vec{U}=(U_t,U_n,U_b)$ is tangential, normal and binormal components of cable structural velocity relative to current velocity $(\vec{V}-\vec{J})$; x, y, z are trail, lateral shift and depth of a point on cable w.r.t. tow-point in the inertial frame; $\theta(s,t)$, $\phi(s,t)$ Euler's angles defining the position of local reference frame $(\vec{t},\vec{n},\vec{b})$ relative to the inertial frame $(\vec{i},\vec{j},\vec{k})$.

Three boundary conditions at the tow-point of the cable are provided by known velocity components of the tow-ship at any time. i.e.

$$\vec{V}(0,t)=\vec{v}(t), \tag{3}$$

Or, in terms of $\vec{y}$, we have

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}\vec{y}(0,t)=\vec{v}(t),$$

At the free end the three boundary conditions can be expressed as

$$\overline{C}\vec{y}(S,t)+\overline{B}(\vec{y}(S,t),t)\dot{\vec{y}}(S,t)+\overline{Q}(\vec{y}(S,t),t)=0, \tag{4}$$

where

$$\overline{C}=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\overline{B}=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_1 & 0 & (m_1V_n-\rho AJ_n)\sin\phi-m_1V_t\cos\phi & 0 \\ 0 & m_1 & 0 & 0 & -(m_1V_b-\rho AJ_b)\sin\phi & m_1V_t \end{bmatrix},$$

$$\overline{Q}=\begin{bmatrix} 0 \\ \frac{1}{2}\rho d(1+eT)^{\frac{1}{2}}C_nU_n(U_n^2+U_t^2)^{\frac{1}{2}}-\rho A\dot{J}_b \\ \frac{1}{2}\rho d(1+eT)^{\frac{1}{2}}C_nU_n(U_n^2+U_t^2)^{\frac{1}{2}}-\rho A\dot{J}_n \end{bmatrix},$$

At $t=0$, it is assumed that the initial condition is known i.e $\vec{y}(s,0)$ is a known function of $s(0\le s\le S)$. This condition along with six boundary conditions provides the complete solution of the governing equations.

Computations start from a steady state solution (more precisely, the tow-ship is assumed to move with constant velocity), which is taken as the initial condition for the whole system.

The variables T and ϕ are determined from equations

$$T'=w\sin\phi+\frac{1}{2}\rho d(1+eT)^{\frac{1}{2}}\pi C_tU_t\,|U_t|$$

$$T\phi' = w\cos\phi + \frac{1}{2}\rho d(1+eT)^{\frac{1}{2}} C_n U_n (U_n^2 + U_b^2)^{\frac{1}{2}}$$

Since $T(S) = 0$, the critical angle $\phi(S)$ must satisfy

$$w\cos\phi + \frac{1}{2}\rho d(1+eT)^{\frac{1}{2}} C_n U_n (U_n^2 + U_b^2)^{\frac{1}{2}} = 0$$

where T' is the partial derivative of T w.r.t. s, holding t fixed; ϕ' partial derivative of ϕ w.r.t. s, holding t fixed.

The position (x, y, z) of the cable, in the inertial frame, are obtained from the relations

$$x' = (1+eT)\cos\theta\cos\phi,$$
$$y' = -(1+eT)\sin\theta\cos\phi,$$
$$z' = -(1+eT)\sin\phi,$$

Integrations in S determine (x, y, z) when eT and the angles θ and ϕ are known.

If we put

$$\dot{r} = V_t\vec{t} + V_n\vec{n} + V_b\vec{b}$$

then V_t, V_n, V_b are obtained from the relations

$$V_t = V_1\cos\theta\cos\phi - V_2\sin\theta\cos\phi - V_3\sin\phi,$$
$$V_n = -V_1\cos\theta\sin\phi + V_2\sin\theta\sin\phi - V_3\cos\phi$$
$$V_b = V_1\sin\theta + V_2\cos\theta$$

where

$$V_1 = \frac{\partial x}{\partial t},\ V_2 = \frac{\partial y}{\partial t},\ V_3 = \frac{\partial z}{\partial t},$$

The angle θ is computed from the relation

$$\theta = \tan^{-1}\left(\frac{V_2}{V_1}\right)$$

J_t, J_n, J_b are computed from the relations

$$J_t = J_1\cos\theta\cos\phi - J_2\sin\theta\cos\phi - J_3\sin\phi,$$
$$J_n = -J_1\cos\theta\sin\phi + J_2\sin\theta\sin\phi - J_3\cos\phi$$
$$J_b = J_1\sin\theta + J_2\cos\theta$$

where J_1, J_2 and J_3 are the current velocities in the inertial frame.

Similarly, U_t, U_n, U_b are computed from the relations

$$U_t = (V_1 - J_1)\cos\theta\cos\phi - (V_2 - J_2)\sin\theta\cos\phi - (V_3 - J_3)\sin\phi,$$
$$U_n = -(V_1 - J_1)\cos\theta\sin\phi + (V_2 - J_2)\sin\theta\sin\phi - (V_3 - J_3)\cos\phi$$
$$U_b = (V_1 - J_1)\sin\theta + (V_2 - J_2)\cos\theta$$

Three components of the tow-ship velocity, free end zero tension along with subsequent two more free end boundary conditions provide a total of six requisite boundary conditions. The solutions of the six governing equations along with six boundary conditions provide dynamic response of the cable array system. The solution procedure starts with the cable array length discretization and time division, followed by writing of finite difference scheme for governing equations at all the nodes and the boundary conditions. All these equations are assembled. The solution of this finite-difference form of the assembly of non-linear algebraic equations is obtained by Newton's method.

3 Numerical Approach

Second order central finite difference method is applied to the governing differential equations to convert them into the algebraic difference approximations.

The total cable-array length S is divided into N segments of arbitrary length

$$0 = s_0 < s_1 < s_2 < \ldots\ldots < s_{N-1} < s_N = S\,.$$

The discrete approximation to $\vec{y}(s_j, t_i)$ is taken to be Y with $Y_j^i \approx \vec{y}(s_j, t_i)$. For convenience the following notations are used.

$$Y^i = (Y_0^i, Y_1^i, \ldots\ldots\ldots, Y_N^i)$$

$$t_{i+\frac{1}{2}} = \frac{1}{2}[t_{i+1} + t_i], \qquad \Delta t = t_{i+1} - t_i,$$

$$s_{j+\frac{1}{2}} = \frac{1}{2}[s_{j+1} + s_j],\ \Delta s = s_{j+1} - s_j,$$

Discretizing the governing equations of motion

$$M\frac{\partial\vec{y}}{\partial s} - N\frac{\partial\vec{y}}{\partial t} - \vec{q} = 0,$$

At the half-grid points $(j+\frac{1}{2}, i+\frac{1}{2})$ and dropping second and higher order terms, we get

$$\frac{1}{2}[M^i_{j+\frac{1}{2}}(\frac{Y^i_{j+1} - Y^i_j}{\Delta s_j}) + M^{i+1}_{j+\frac{1}{2}}(\frac{Y^{i+1}_{j+1} - Y^i_j}{\Delta s_j})]$$
$$-\frac{1}{2}[N_j^{i+\frac{1}{2}}(\frac{Y^i_{j+1} - Y^i_j}{\Delta t_i}) + N_{j+1}^{i+\frac{1}{2}}(\frac{Y^{i+1}_{j+1} - Y^i_j}{\Delta t_i})]$$
$$-\frac{1}{2}[q^i_{j+\frac{1}{2}} + q^{i+1}_{j+\frac{1}{2}}] = 0, \tag{5}$$

Denoting the LHS of the equation (5) by ϕ^{i+1}_{j+1}, we get the difference approximation

$$\Phi^{i+\frac{1}{2}}_{j+\frac{1}{2}} = 0,\ \text{j=0,1,.......,N-1;} \tag{6}$$

to the governing equations.

Similarly, the boundary conditions are approximated by

$$\Phi_0^{i+\frac{1}{2}} = DY_0^{i+1} = \vec{v}(t_{i+1}), \tag{7}$$

$$\Phi_N^{i+\frac{1}{2}} = CY_N^{i+1} + B(Y_N^{i+\frac{1}{2}}, t_{i+\frac{1}{2}})[\frac{Y_N^{i+1} - Y_N^i}{\Delta t_i}] + Q(Y_N^{i+\frac{1}{2}}, t_{i+\frac{1}{2}}) = 0, \tag{8}$$

The three equations (6), (7) and (8) can be written together as

$$\Phi^{i+\frac{1}{2}}(Y^{i+1}, Y^i) = 0, \tag{9}$$

where

$$\Phi^{i+\frac{1}{2}} = (\Phi_0^{i+\frac{1}{2}}, \Phi_{\frac{1}{2}}^{i+\frac{1}{2}}, \Phi_{\frac{3}{2}}^{i+\frac{1}{2}}, ..., \Phi_{N-\frac{3}{2}}^{i+\frac{1}{2}}, \Phi_{N-\frac{1}{2}}^{i+\frac{1}{2}}, \Phi_N^{i+\frac{1}{2}}),$$

The system (9) is an implicit, centered, second order approximation to the system of hyperbolic pdes. Given Y^i at time t_i, the system of equations (9) determine Y^{i+1} at t_{i+1}. Further we assume that the initial state of the cable Y^0 is known.

The non-linear algebraic equations (9) are solved by iteration in the time domain using Newton's method. The precise algorithm of Newton's method is given below.

(a) Obtain an estimate for Y^{i+1} by extrapolating Y^{i-1} and Y^i, i.e.

$$Y^{i+1} = Y^i + \Delta t[\frac{Y^{i-1} - Y^i}{\Delta t_{i-1}}]$$

(b) Compute a correction to Y^{i+1} by solving the linear system

$$\dot{\Phi}^{i+\frac{1}{2}} \Delta Y = -\Phi^{i+\frac{1}{2}}$$

where

$$\dot{\Phi}^{i+\frac{1}{2}} = \frac{\partial \Phi^{i+\frac{1}{2}}}{\partial Y^{i+1}}$$

is the Jacobian of the above system.

(c) $Y^{i+1} = Y^i + \Delta Y$
gives the improved value of Y^{i+1}.

(d) If the absolute value of maximum relative change in any component of the solution Y^{i+1} is less than 10^{-3}, increment the time and go to step (a), otherwise repeat (b) and (c) using new value of Y^{i+1}.

The iteration is observed to converge quadratically. In step (a), using Y^i as the initial guess for Y^{i+1} is sufficient to achieve convergence. Gauss elimination with partial pivoting is applied to solve the linear system.

4 Result and Discussions

The developed Newton's scheme is implemented on the underwater towed cable-array model. The underwater towed cable-array system is discussed under a six segment model, in which dynamic analysis of towed cable is discussed. In the towed cable-array model, the steady ocean current (0.5 m/s) is taken.

4.1 Six Segment Cable model

Here we discuss unsteady-state behavior of the cable during the ship manoeuvring for three different oceanic current conditions, using the developed code. Fig 4 illustrates the towed array system while Table 1 gives the physical characteristics of each segment of six segment cable model.

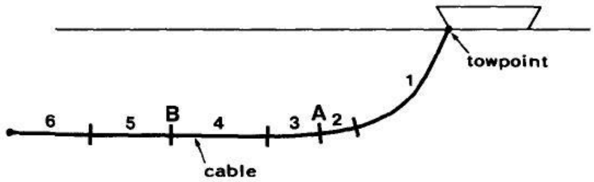

Fig 4: Six Segment Towed Array System.

Table 1: Tow Cable System Physical Properties.

Segment	Length (m)	Diamter (m)	Weight (N/m)	C_t	C_n
1	723.0	0.04060	2.3349	0.01500	2.0
2	8.23	0.079375	0.0	0.00898	1.8
3	71.02	0.079375	0.0	0.00898	1.8
4	156.36	0.079375	0.0	0.00898	1.8
5	38.71	0.079375	0.0	0.00898	1.8
6	30.48	0.025400	0.569134	0.02168	1.8

4.2 Dynamic state analysis

Dynamic state analysis of underwater towed-cable is discussed when tow-ship changes its speed in a fixed direction (i.e., transient behavior) making linear profile. Computations start from a steady state solution which is taken as the initial condition for the dynamic state solution.

Linear Profile

Here we assume that the tow-ship changes its speed along the x-direction in a straight line path. This means that the tow-ship changes its speed along a straight line path in each time step. The linear profile is given by

$$v = (v_f - v_0)(t/T) + v_0, \tag{10}$$

where v_0 and v_f are the tow-ship's initial and final speeds, respectively, and T is the elapsed time taken to reach the final speed.

Case (i): When tow-ship accelerates from 4 to 12 m/s in a linear profile

Fig 5 shows the graph between the trail and the cable depth when the tow-ship accelerates from 4 to 12 m/s in a linear profile, when there is no current, against and along the current directions respectively. The cable depth varies from 38.5 to 38.6 m, 33.6 to 33.8 m and 44.4 to 44.5 m, when the tow-ship accelerates, when there is no current, against and along the current directions, respectively as shown in Fig 5 (a), (b), (c). Thus for three different current situations, the cable depth varies in the range 33.6 to 44.5 m when the tow-ship accelerates from 4 to 12 m/s along the x-direction. It is observed that maximum cable depth occurs when the tow-ship accelerates along the current direction and minimum cable depth occurs when the tow-ship accelerates against the current direction.

Fig 6 shows the graph between cable length and tension when the tow-ship accelerates from 4 to 12 m/s in a linear profile, (a) when there is no current, (b) against current direction and (c) along the current directions, respectively. The tow-point tension varies from 17.0 to 82.8 kN, 21.5 to 89.8 kN and 13.1 to 76.3 kN, when the tow-ship accelerates, when there is no current, against and along the current directions, respectively as shown in Fig 6 (a), (b) and (c). Thus for three different current situations, the tow-point tension varies in the range 13.1 to 89.8 kN when the tow-ship accelerates from 4 to 12 m/s along the x-direction. It is observed that maximum tow-point tension occurs when the tow-ship accelerates against the current direction and minimum tow-point tension occurs when the tow-ship accelerates along the current direction.

Case (ii): When tow-ship decelerates from 12 to 4 m/s in linear profile

Fig 7 shows the graph between the trail and the cable depth when the tow-ship decelerates from 12 to 4 m/s in a linear profile, when there is no current, against and along the current directions, respectively. The cable depth varies from 6.7 to 6.9 m, 5.9 to 6.1 m and 7.7 to 7.9 m, when the tow-ship decelerates, when there is no current, against and along the current directions, respectively as shown in Fig 7 (a), (b) and (c). Thus for three different current situations, the cable depth varies in the range 5.9 to 7.9 m when the tow-ship decelerates from 12 to 4 m/s along the x-direction. It is observed that maximum cable depth occurs when the tow-ship decelerates along the current direction and minimum cable depth occurs when the tow-ship decelerates against the current direction.

Fig 8 shows the graph between cable length and tension when the tow-ship decelerates from 12 to 4 m/s in a linear profile, when there is no current, against and along the current directions, respectively. The tow-point tension varies from 58.1 to 153.2 kN, 68.3 to 166.3 kN and 48.6 to 140.7 kN, when the tow-ship decelerates, when there is no current, against and along the current directions, respectively as shown in Fig 8 (a), (b) and (c). Thus for three different current situations, the tow-point tension variesin the range 48.6 to 166.3 kN when the tow-ship decelerates from 12 to 4 m/s along the x-direction. It is observed that maximum tow-point tension occurs when the tow-ship decelerates against the current direction and minimum tow-point tension occurs when the tow-ship decelerates along the current direction.

Case (iii): When tow-ship accelerates from 4 to 12 m/s thereafter decelerates from 12 to 4 m/s in linear profile

Fig 9 shows the graph between the trail and the cable depth when the tow-ship accelerates from 4 to 12 m/s thereafter decelerates from 12 to 4 m/s in a linear profile, when there is no current, against and along the current directions, respectively. The cable depth varies from 38.4 to 38.5 m, 33.6 to 33.8 m and 44.4 to 44.5 m, when the tow-ship accelerates and decelerates, when there is no current, against and along the current directions, respectively as shown in Fig 9 (a), (b) and (c). Thus for three different current situations, the cable depth varies in the range 33.6 to 44.5 m when the tow-ship accelerates from 4 to 12 m/s and then decelerates from 12 to 4 m/s along the x-direction. It is observed that maximum cable depth occurs when the tow-ship accelerates and decelerates along the current direction and minimum cable depth occurs when the tow-ship is accelerating and decelerating against the current direction.

Fig 10 shows the graph between the cable length and tension when the tow-ship accelerates from 4 to 12 m/s thereafter decelerates from 12 to 4 m/sec in a linear profile, when there is no current, against and along the current directions, respectively. The tow-point tension varies from 17.0 to 40.9 kN, 21.5 to 48.2 kN and 13.1 to 33.9 kN, when the tow-ship accelerates and decelerates, when there is no current, against and along the current directions respectively as shown in Fig 10 (a), (b) and (c). Thus for three different current situations, the tow-point tension varies in the range 13.1 to 48.2 kN when the tow-ship accelerates from 4 to 12 m/s and then decelerates from 12 to 4 m/s along the x-direction. It is observed that maximum tow-point tension occurs when the tow-ship accelerates and decelerates against the current direction and minimum tow-point tension occurs when the tow-ship accelerates and decelerates along the current direction.

Table 2: Cable depth Range (m) in Linear Profile.

Speed	Linear Profile		
	$J_1 = 0$	$J_1 = -0.5$	$J_1 = 0.5$
4 - 12 m/s	38.5 - 38.6	33.6 - 33.8	44.4 - 44.5
12 - 4 m/s	6.7 - 6.9	5.9 - 6.1	7.7 - 7.9
4 - 12m/s & 12 - 4 m/s	38.4 - 38.5	33.6 - 33.8	44.4 - 44.5

Table 3: Tow-point Tension Range (kN) in Linear Profile.

Speed	Linear Profile		
	$J_1 = 0$	$J_1 = -0.5$	$J_1 = 0.5$
4 - 12 m/s	17.0 - 82.8	21.5 - 89.8	13.1 - 76.3
12 - 4 m/s	76.3 - 153.2	68.3 -166.3	48.6 -140.7
4 - 12 m/s & 12 - 4 m/s	17.0 -40.9	21.5 - 48.2	13.1 -33.9

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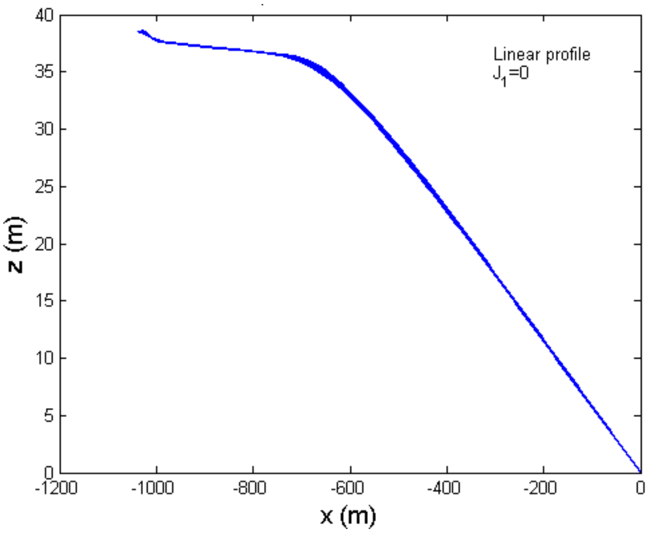
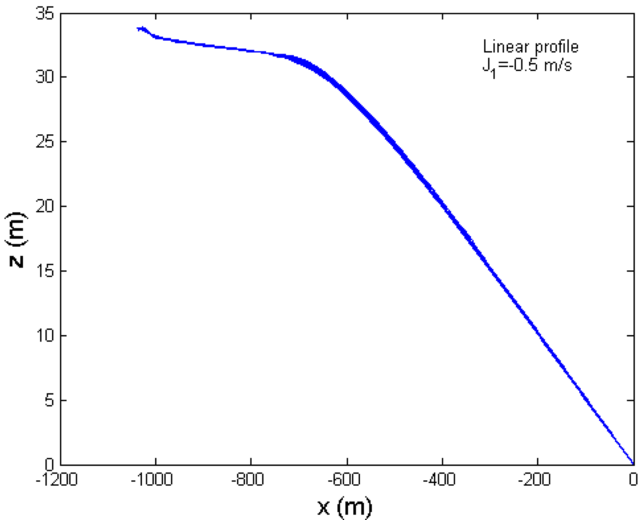
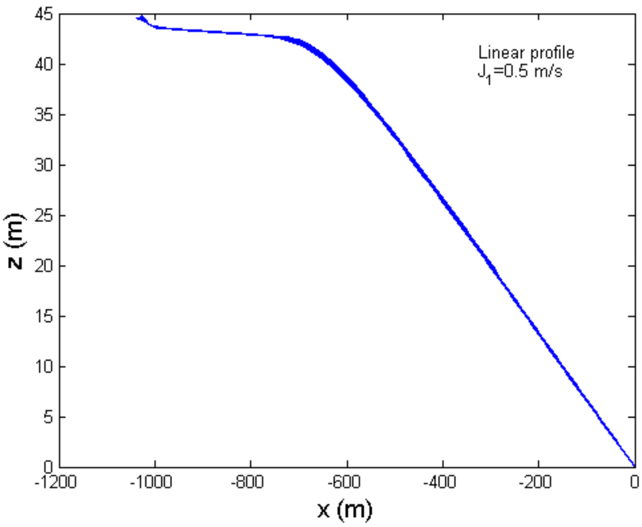


(a) When there is no current.

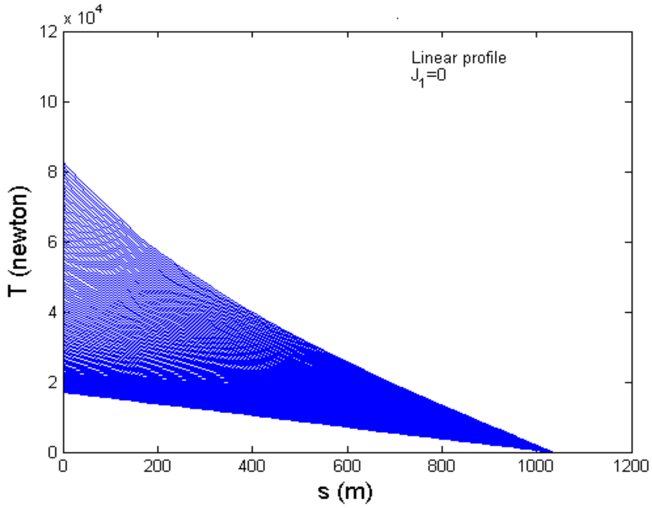


(b) Against the current direction.

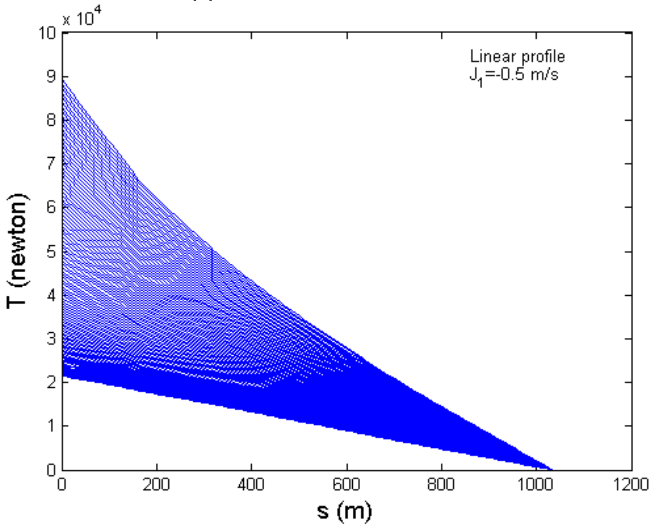


(c) Along the current direction.

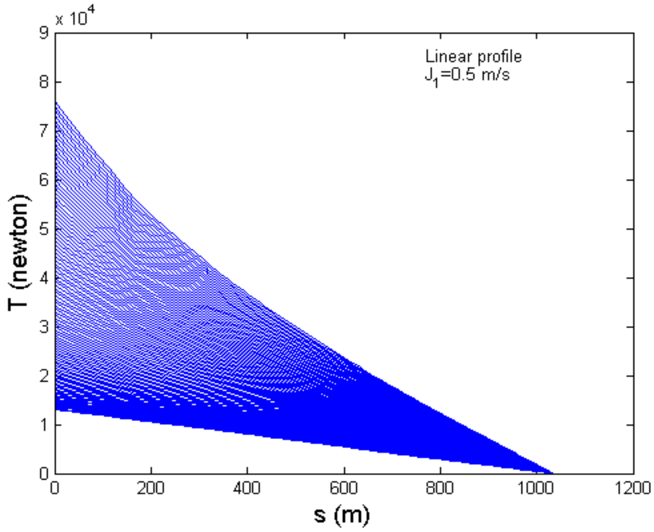
Fig 5: Graph between the Trail and the Cable Depth when the Tow-Ship Accelerates from 4 to 12 m/s in Linear Profile: (a), (b), (c).



(a) When there is no current.



(b) Against the current direction.



(c) Along the current direction.

Fig 6: Graph between the Cable length and Tension when the Tow-Ship Accelerates from 4 to 12 m/s in Linear Prfile: (a), (b), (c).

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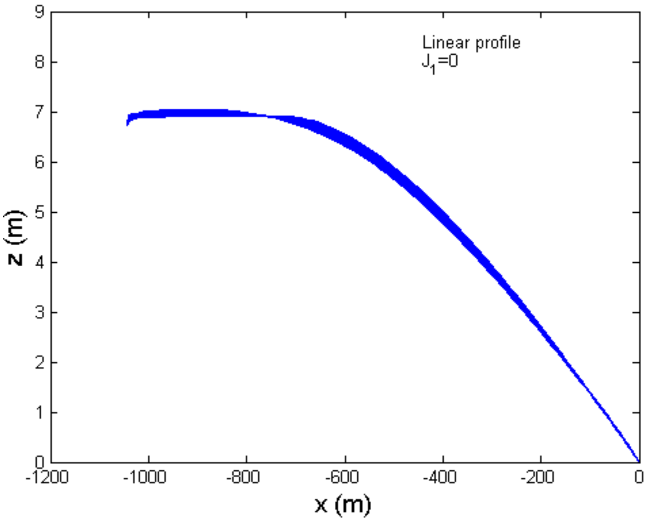
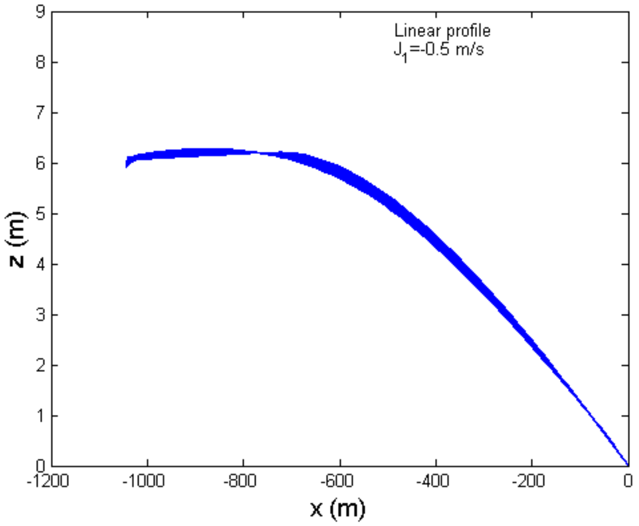
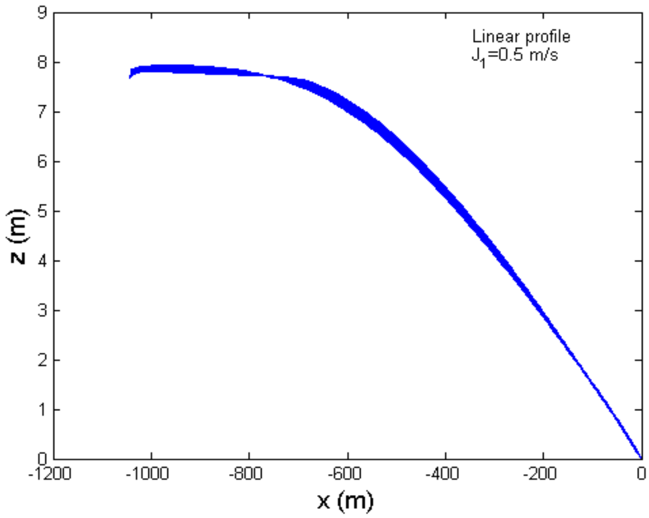


(a) When there is no current.

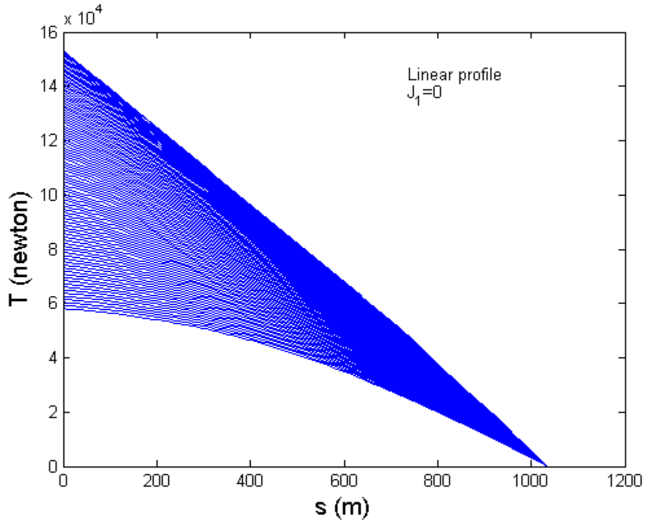


(b) Against the current direction.

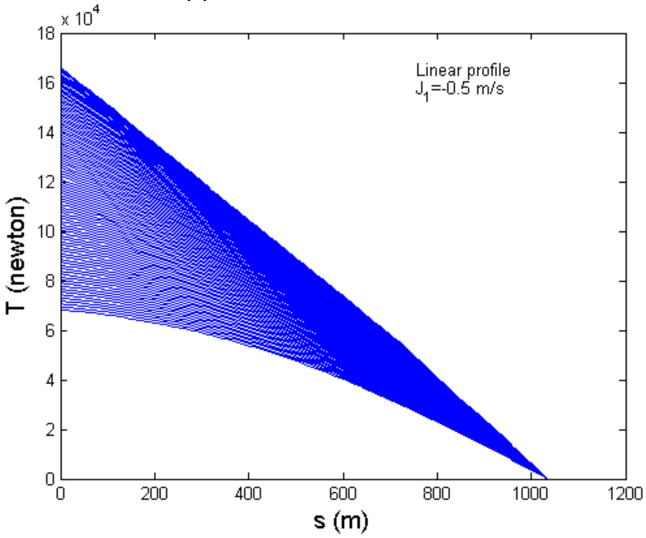


(c) Along the current direction.

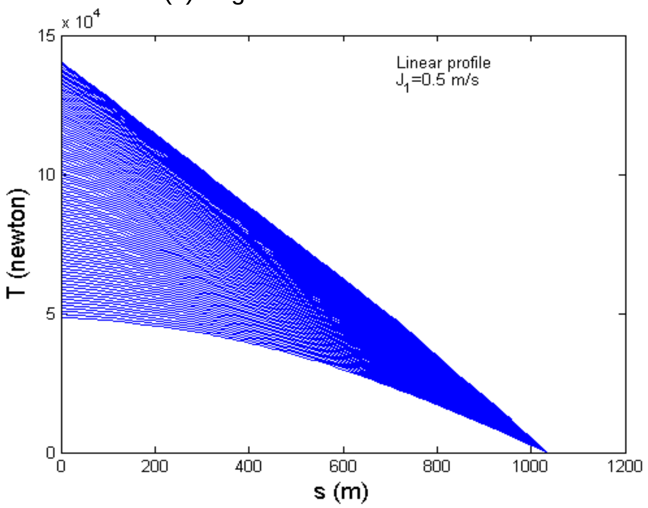
Fig 7: Graph between the Trail and the Cable Depth when the Tow-Ship Decelerates from 12 to 4 m / s in Linear Profile: (a), (b), (c).



(a) When there is no current.

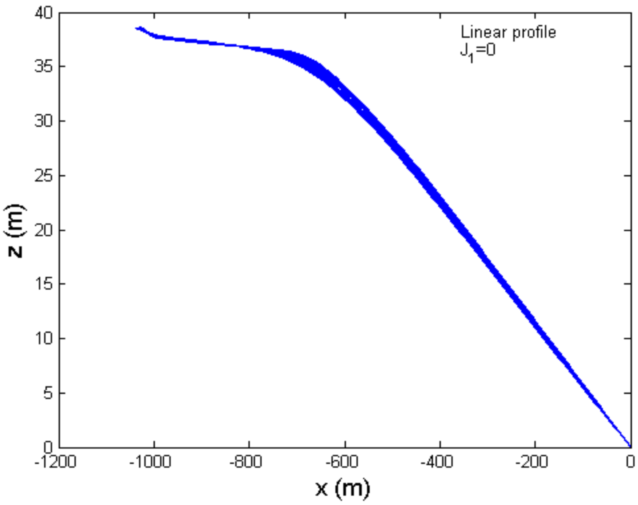


(b) Against the current direction.

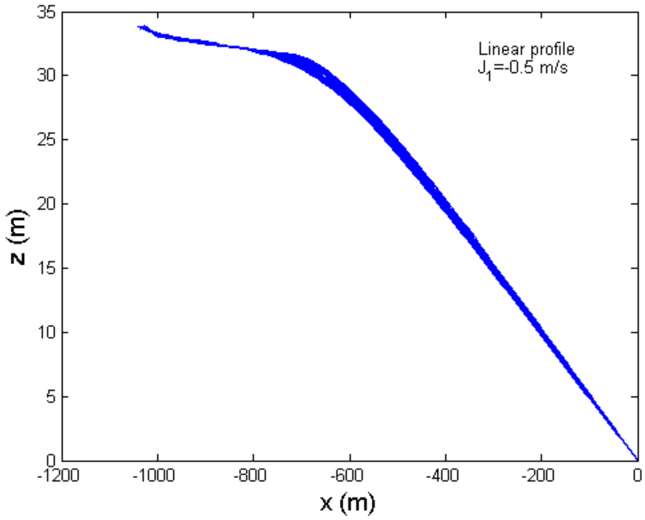


(c) Along the current direction.

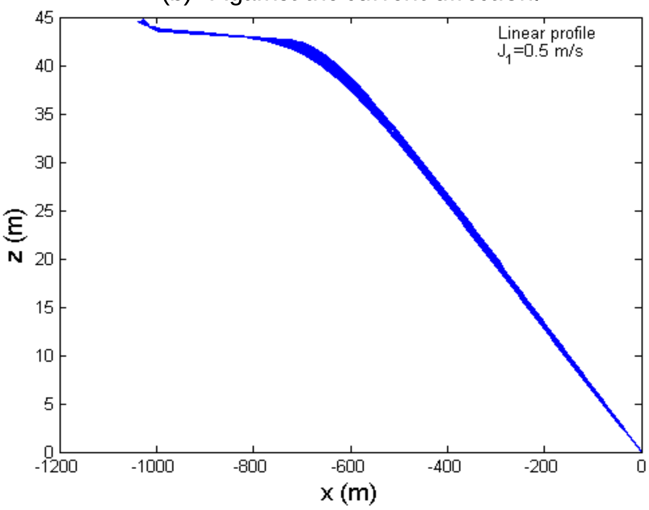
Fig 8: Graph between the Cable length and Tension when the Tow-Ship Decelerates from 12 m / s to 4 m / s in Linear Profile: (a), (b), (c):



(a) When there is no current.

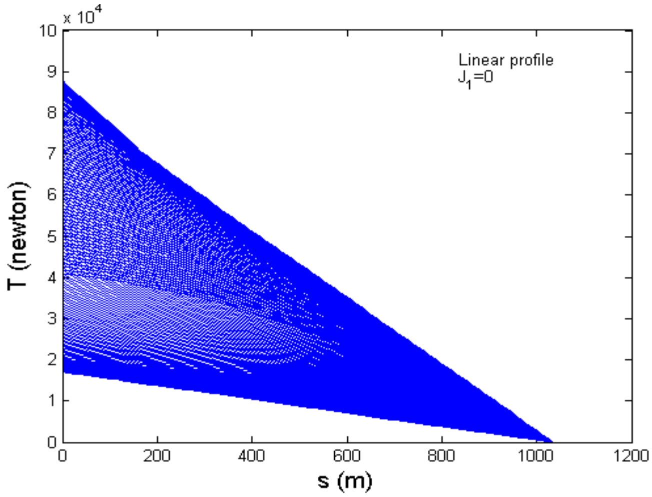


(b) Against the current direction.

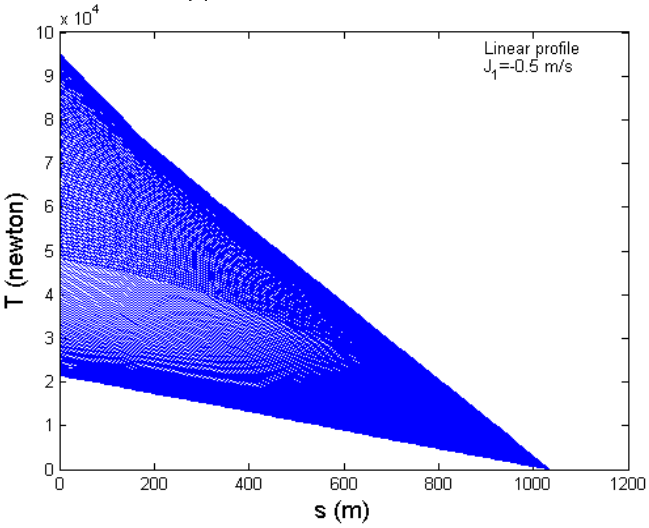


(c) Along the current direction.

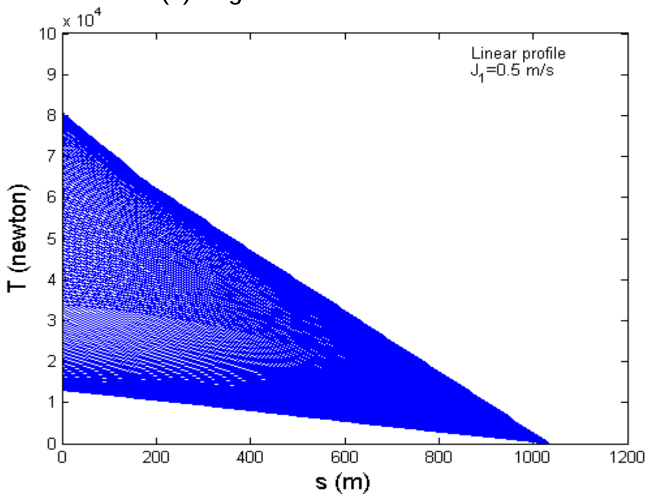
Fig 9: Graph between the Trail and the Cable Depth when the Tow-Ship Accelerates from 4 to 12 m / s then Decelerates from 12m / s to 4 m / s in Linear Profile: (a),(b),(c).



(a) When there is no current.



(b) Against the current direction.



(c) Along the current direction.

Fig 10: Graph between the Cable length and Tension when the Tow-Ship Accelerates from 4 to 12 m / s then Decelerates from 12m / s to 4 m / s in Linear Profile: (a),(b),(c).

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5 Conclusion

In this study, a three-dimensional numerical program is developed for the analysis of the underwater towed cable-array system when tow-ship makes linear profile during manouring. An implicit finite difference method is employed for solving the three dimensional cable equations. In order to solve the non-linear and coupled problems, Newton's iteration scheme is used, and satisfactory results are obtained. The developed numerical program can be applied to towed array systems for detecting a moving object or submarine.

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